

Varying Alpha

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Abstract

We review properties of cosmological theories for the variation of the fine structure 'constant'. We highlight some general features of the cosmological models that exist in these theories with reference to recent quasar data that are consistent with time-variation in the fine structure 'constant' since a redshift of 3.5.

1 Introduction

There are several reasons why the possibility of varying constants should be taken seriously [1]. First, we know that the best candidates for unification of the forces of nature in a quantum gravitational environment only seem to exist in finite form if there are many more dimensions of space than the three that we are familiar with. This means that the true constants of nature are defined in higher dimensions and the three-dimensional shadows we observe are no longer fundamental and need not be constant. Any slow change in the scale of the extra dimensions would be revealed by measurable changes in our three-dimensional 'constants'. Second, we appreciate that some apparent constant might be determined partially or completely by spontaneous symmetry-breaking processes in the very early universe. This introduces an irreducibly random element into the values of those constants. They may be different in different parts of the universe. The most dramatic manifestation of this process is provided by the chaotic and eternal inflationary universe scenarios where symmetries determining both the number and the strength of forces in the universe at low energy can break differently in different regions. Third, any outcome of a theory of quantum gravity will be intrinsically probabilistic. It is often imagined that the probability distributions for observables will be very sharply peaked but this may not be the case for all possibilities. Fourth, a non-uniqueness of the vacuum state for the universe would allow other numerical combinations of the

constants to have occurred in different places. String theory indicates that there is a huge 'landscape' ($> 10^{500}$) of possible vacuum states that the universe can find itself residing in as it expand and cools. Each will have different constants and associated forces and symmetries. It is sobering to remember that at present we have no idea why any of the constants of Nature take the numerical values they do and we have never successfully predicted the value of any dimensionless constant in advance of its measurement. However, the last reason to consider varying constants is currently the most compelling. For the first time there is a body of detailed astronomical evidence for the time variation of a traditional constant. The observational programme of Webb et al [2, 3] has completed detailed analyses of three separate quasar absorption line data sets taken at Keck and finds persistent evidence consistent with the fine structure constant, α , having been *smaller* in the past, at $z = 1 - 3.5$. The shift in the value of α for all the data sets is given provisionally by $\Delta\alpha/\alpha = (-0.57 \pm 0.10) \times 10^{-5}$. This result is currently the subject of detailed analysis and reanalysis by the observers in order to search for possible systematic biases in the astrophysical environment or in the laboratory determinations of the spectral lines. So far it has not been undermined or confirmed by other observations (for the most reason discussion of the status of uncertainties, see [4]).

The first investigations of time-varying constants were those made by Lord Kelvin and others interested in possible time-variation of the speed of light at the end of the nineteenth century. In 1935 Milne devised a theory of gravity, of a form that we would now term 'bimetric', in which there were two times – one (t) for atomic phenomena, one (τ) for gravitational phenomena – linked by $\tau = \log(t/t_0)$. Milne [5] required that the 'mass of the universe' (what we would now call the mass inside the particle horizon $M \approx c^3 G^{-1} t$) be constant. This required $G \propto t$. Interestingly, in 1937 the biologist J.B.S. Haldane took a strong interest in this theory and wrote several papers [6] exploring its consequences for the evolution of life. He argued that biochemical activation energies might appear constant on the t timescale yet increase on the τ timescale, giving rise to a non-uniformity in the evolutionary process. Also at this time there was widespread familiarity with the mysterious 'large numbers' $O(10^{40})$ and $O(10^{80})$ through the work of Eddington (although they had first been noticed by Weyl [7] – see ref. [8] and [1] for the history). These two ingredients were merged by Dirac in 1937 in a famous development (supposedly written on his honeymoon) that proposed that these large numbers (10^{40}) were actually equal, up to small dimensionless factors. Thus, if we form $N \sim c^3 t / G m_n \sim 10^{80}$, the number of nucleons in the visible universe, and equate it to the square of $N_1 \sim e^2 / G m_n^2 \sim 10^{40}$, the ratio of the electrostatic and gravitational forces between two protons then we are led to conclude that one of the constants, e, G, c, h, m_n must vary with time. Dirac [9] chose $G \propto t^{-1}$ to carry the time variation. Unfortunately, this hypothesis did not survive very long. Edward Teller [10] pointed out that such a steep increase in G to the past led to huge increases in the Earth's surface temperature in the past. The luminosity of the sun varies as $L \propto G^7$ and the radius of the Earth's orbit as $R \propto G^{-1}$ so the Earth's surface temperature T_\oplus varies as $(L/R^2)^{1/4} \propto G^{9/4} \propto t^{-9/4}$ and

would exceed the boiling point of water in the pre-Cambrian era. Life would be eliminated. Gamow subsequently suggested that the time variation needed to reconcile the large number coincidences be carried by e rather than G , but again this strong variation was soon shown to be in conflict with geophysical and radioactive decay data. This chapter was brought to an end by Dicke [11] who pointed out that the $N \sim N_1^2$ large number coincidence was just the statement that t , the present age of the universe when our observations are being made, is of order the main-sequence stellar lifetime, $t_{ms} \sim (Gm_n^2/hc)^{-1}h/m_n c^2 \sim 10^{10}$ yrs, and therefore inevitable for observers made out of chemical elements heavier than hydrogen and helium. Dirac never accepted this anthropic explanation for the large number coincidences (believing that 'observers' would be present in the universe long after the stars had died) but curiously can be found making exactly the same type of anthropic argument to defend his own varying G theory by highly improbable arguments (that the Sun accretes material periodically during its orbit of the galaxy and this extra material cancels out the effects of overheating in the past) in unpublished correspondence with Gamow in 1967 (see [1] for fuller details). Dirac's biographer has revealed that in 1993 he expressed 'an article of faith... that the human race will continue to live for ever and will develop and progress without limit' [12]. This belief motivates his comments relating to the anthropic argument.

Dirac's proposal acted as a stimulus to theorists, like Jordan, Brans and Dicke [13], to develop rigorous theories which included the time variation of G self-consistently, by modelling it as arising from the space-time variation of some scalar field $\phi(\mathbf{x}, t)$ whose motion both conserved energy and momentum and created its own gravitational field variations. The possibility that α varies in time has led to the first extensive exploration of simple self-consistent theories in which α changes occur through the dynamics of some scalar field.

2 A Simple Varying-Alpha Theory

We consider some of the cosmological consequences of a simple theory of time varying α . Such a theory was first formulated by Bekenstein [14] as a generalisation of Maxwell's equations but ignoring the consequences for the gravitational field equations. We completed this theory [15] to include the coupling to the gravitational sector and analysed its general cosmological consequences and it is referred to as the Bekenstein-Sandvik-Barrow-Magueijo (BSBM) theory below. Extensions to include the weak interaction via a generalised Weinberg-Salam theory have also been explored, [16, 17]. Extensions to include the weak interaction via a generalised Weinberg-Salam theory have also been explored, [16, 17].

The idea that the charge on the electron, or the fine structure constant, might vary in cosmological time was proposed in 1948 by Teller, [10], who suggested that $\alpha \propto (\ln t)^{-1}$ was implied by Dirac's proposal that $G \propto t^{-1}$ and the numerical coincidence that $\alpha^{-1} \sim \ln(hc/Gm_p^2)$, where m_p is the proton mass. Later, in 1967, Gamow [18] suggested $\alpha \propto t$ as an alternative to Dirac's time-variation of the gravitation constant, G , as a solution of the large numbers

coincidences problem; and in 1963 Stanyukovich had also considered varying α in this context [19]. However, any such power-law variation in the recent past was soon ruled out by other geological evidence [20].

There are a number of possible theories allowing for the variation of the fine structure constant, α . In the simplest cases one takes c and \hbar to be constants and attributes variations in α to changes in e or the permittivity of free space (see [21] for a discussion of the meaning of this choice). Thus $e_0 \rightarrow e = e_0 \epsilon(x^\mu)$, where ϵ is a dimensionless scalar field and e_0 is a constant denoting the present value of e . This implies that some well established assumptions, like charge conservation, must give way [22]. Nevertheless, local gauge invariance and causality are maintained.

Since e is the electromagnetic coupling, the ϵ field couples to the gauge field as ϵA_μ in the Lagrangian and the gauge transformation which leaves the action invariant is $\epsilon A_\mu \rightarrow \epsilon A_\mu + \chi_{,\mu}$, rather than the usual $A_\mu \rightarrow A_\mu + \chi_{,\mu}$. The gauge-invariant electromagnetic field tensor is therefore

$$F_{\mu\nu} = \frac{1}{\epsilon} ((\epsilon A_\nu)_{,\mu} - (\epsilon A_\mu)_{,\nu}), \quad (1)$$

which reduces to the usual form when ϵ is constant. The electromagnetic part of the action is still

$$S_{em} = - \int d^4x \sqrt{-g} F^{\mu\nu} F_{\mu\nu}. \quad (2)$$

and the dynamics of the ϵ field are controlled by the kinetic term

$$S_\epsilon = - \frac{1}{2} \frac{\hbar}{l^2} \int d^4x \sqrt{-g} \frac{\epsilon_{,\mu} \epsilon^{,\mu}}{\epsilon^2}, \quad (3)$$

as in dilaton theories. Here, l is the characteristic length scale of the theory, introduced for dimensional reasons. This constant length scale gives the scale down to which the electric field around a point charge is accurately Coulombic. The corresponding energy scale, $\hbar c/l$, has to lie between a few tens of MeV and Planck scale, $\sim 10^{19} \text{ GeV}$ to avoid conflict with experiment.

The field equations are

$$G_{\mu\nu} = 8\pi G (T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^\psi + T_{\mu\nu}^{\text{em}} e^{-2\psi}). \quad (4)$$

The stress tensor of the ψ field is derived from the lagrangian $\mathcal{L}_\psi = -\frac{\omega}{2} \partial_\mu \psi \partial^\mu \psi$ and the ψ field obeys the equation of motion

$$\square \psi = \frac{2}{\omega} e^{-2\psi} \mathcal{L}_{\text{em}} \quad (5)$$

where we have defined the coupling constant $\omega = (c)/l^2$. This constant is of order ~ 1 if, as in [15], the energy scale is similar to Planck scale. It is clear that \mathcal{L}_{em} vanishes for a sea of pure radiation since then $\mathcal{L}_{\text{em}} = (E^2 - B^2)/2 = 0$. We therefore expect the variation in α to be driven by electrostatic and magnetostatic energy-components rather than electromagnetic radiation.

In order to make quantitative predictions we need to know how much of the non-relativistic matter contributes to the RHS of Eqn. (5). This is parametrised by $\zeta \equiv \mathcal{L}_{\text{em}}/\rho$, where ρ is the energy density, and for baryonic matter $\mathcal{L}_{\text{em}} = E^2/2$. For protons and neutrons ζ_{p} and ζ_{n} can be *estimated* from the electromagnetic corrections to the nucleon mass, 0.63 MeV and -0.13 MeV, respectively [23]. This correction contains the $E^2/2$ contribution (always positive), but also terms of the form $j_{\mu}a^{\mu}$ (where j_{μ} is the quarks' current) and so cannot be used directly. Hence we take a guiding value $\zeta_{\text{p}} \approx \zeta_{\text{n}} \sim 10^{-4}$. Furthermore the cosmological value of ζ (denoted ζ_{m}) has to be weighted by the fraction of matter that is non-baryonic. Hence, ζ_{m} depends strongly on the nature of the dark matter and can take both positive and negative values depending on which of Coulomb-energy or magnetostatic energy dominates the dark matter of the Universe. It could be that $\zeta_{\text{CDM}} \approx -1$ (as for superconducting cosmic strings, with $\mathcal{L}_{\text{em}} \approx -B^2/2$), or $\zeta_{\text{CDM}} \ll 1$ (neutrinos). BBN predicts an approximate value for the baryon density of $\Omega_{\text{B}} \approx 0.03$ (where Ω_{B} is the density of matter in units of the critical density $3H^2/8\pi G$) with a Hubble parameter of $H = 60 \text{ Kms}^{-1} \text{ Mpc}^{-1}$, implying $\Omega_{\text{CDM}} \approx 0.3$. Thus, depending on the nature of the dark matter, ζ_{m} can be virtually any number between -1 and $+1$. The uncertainties in the underlying quark physics and especially the constituents of the dark matter make it difficult to impose more certain bounds on ζ_{m} .

2.1 The cosmological equations

Assuming a homogeneous and isotropic Friedmann metric with expansion scale factor $a(t)$ and curvature parameter k in eqn. (4), we obtain the field equations ($c \equiv 1$)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left(\rho_{\text{m}} (1 + \zeta_{\text{m}} \exp[-2\psi]) + \rho_{\text{r}} \exp[-2\psi] + \frac{\omega}{2} \dot{\psi}^2 \right) - \frac{k}{a^2} + \frac{\Lambda}{3} \quad (6)$$

where Λ is the cosmological constant. The scalar field obeys

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{2}{\omega} \exp[-2\psi] \zeta_{\text{m}} \rho_{\text{m}}, \quad (7)$$

where $H \equiv \dot{a}/a$ is the Hubble rate. We can rewrite this as

$$(\dot{\psi} a^3) = N \exp[-2\psi] \quad (8)$$

where N is a positive constant defined by $N = -2\zeta_{\text{m}}\rho_{\text{m}}a^3/\omega$. Note that the sign of the evolution of ψ is dependent on the sign of ζ_{m} . Since the observational data is consistent with a *smaller* value of α in the past, we will in this paper confine our study to *negative* values of ζ_{m} , in line with our recent discussion in Refs. [15]. The conservation equations for the non-interacting radiation and matter densities give $\rho_{\text{m}} \propto a^{-3}$ and $\rho_{\text{r}} e^{-2\psi} \propto a^{-4}$, respectively. This theory enables the cosmological consequences of varying e , to be analysed self-consistently rather than by changing the constant value of e in the standard theory to another constant value, as in the original proposals made in response

to the large numbers coincidences. We shall consider the form of the solutions to these equations when the universe is successively dominated by the kinetic energy of the scalar field ψ , pressure-free matter, radiation, negative spatial curvature, and positive cosmological constant. Our analytic expressions are checked by numerical solutions of (6) and (7). There are a number of conclusions that can be drawn from the study of the simple BSBM models with $\zeta_m < 0$. These models give a good fit to the varying α implied by the QSO data of refs. [2, 3]. There is just a single parameter to fit and this is given by the choice

$$-\frac{\zeta_m}{\omega} = (2 \pm 1) \times 10^{-4} \quad (9)$$

The simple solutions predict a slow (logarithmic) time increase during the dust era of $k = 0$ Friedmann universes. The cosmological constant turns off the time-variation of α at the redshift when the universe begins to accelerate ($z \sim 0.7$) and so there is no conflict between the α variation seen in quasars at $z \sim 1 - 3.5$ and the limits on possible variation of α deduced from the operation of the Oklo natural reactor [24] (even assuming that the cosmological variation applies unchanged to the terrestrial environment). The reactor operated 1.8 billion years ago at a redshift of only $z \sim 0.1$ when no significant variations were occurring in α . The slow logarithmic increase in α also means that we would not expect to have seen any effect yet in the anisotropy of the microwave backgrounds [25, 26]: the value of α at the last scattering redshift, $z = 1000$, is only 0.005% lower than its value today. Similarly, the essentially constant evolution of α predicted during the radiation era leads us to expect no measurable effects on the products of Big Bang nucleosynthesis (BBN) because α was only 0.007% smaller at BBN than it is today.

Theories in which α varies will in general lead to violations of the weak equivalence principle (WEP). This is because the α variation is carried by a field like ψ and this couples differently to different nuclei because they contain different numbers of electrically charged particles (protons). The theory discussed here has the interesting consequence of leading to a relative acceleration of order 10^{-13} [27] if the free coupling parameter is fixed to the value given in eq. (9) by using a best fit of the theories cosmological model to the QSO observations of refs. [2, 3]. Other predictions of such WEP violations have also been made in refs. [28, 23, 29, 30]. The observational upper bound on this parameter from direct experiment is just an order of magnitude larger, at 10^{-12} , and limits from the motion of the Moon are of similar order, but space-based tests planned for the STEP mission [31] are expected to achieve a sensitivity of order 10^{-18} and will provide a completely independent check on theories of time-varying e and α . This is an exciting prospect for the future.

2.2 The nature of the Friedmann solutions

Let us present the predicted cosmological evolution of α in the BSBM theory, that we summarised above, in a little more detail. During the radiation era the expansion scale factor of the universe increases as $a(t) \sim t^{1/2}$ and α is

essentially constant in universes with an entropy per baryon and present value of α like our own. It increases in the dust era, where $a(t) \sim t^{2/3}$. The increase in α however, is very slow and $\alpha \sim 2N \log(t/t_1)$. This slow increase continues until the expansion becomes dominated by negative curvature, $a(t) \sim t$, or by a cosmological vacuum energy, $a(t) \sim \exp[\Lambda t/3]$. Thereafter α asymptotes rapidly to a constant. If we set the cosmological constant equal to zero and $k = 0$ then, during the dust era, α would continue to increase indefinitely. The effect of the expansion is very significant at all times. Non-zero curvature or a cosmological constant stops the increase in the value of α that occurs during the dust-dominated era. Hence, if the spatial curvature and Λ are both too *small* it is possible for the fine structure constant to grow too large for biologically important atoms and nuclei to exist in the universe. There will be a time in the future when α reaches too large a value for life to emerge or persist. The closer a universe is to flatness or the closer Λ is to zero so the longer the monotonic increase in α will continue, and the more likely it becomes that life will be extinguished. Conversely, a non-zero positive Λ or a non-zero negative curvature will stop the increase of α earlier and allow life to persist for longer. If life can survive into the curvature or Λ -dominated phases of the universe's history then it will not be threatened by the steady cosmological increase in α unless the universe collapses back to high density.

There have been several studies, following Carter, [32] of the need for life-supporting universes to expand close to the 'flat' Einstein de Sitter trajectory for long periods of time. This ensures that the universe cannot collapse back to high density before galaxies, stars, and biochemical elements can form by gravitational instability, or expand too fast for stars and galaxies to form by gravitational instability [33, 8]. Likewise, it was pointed out by Barrow and Tipler, [8] that there are similar anthropic restrictions on the magnitude of any cosmological constant, Λ . If it is too large in magnitude it will either precipitate premature collapse back to high density (if $\Lambda < 0$) or prevent the gravitational condensation of any stars and galaxies (if $\Lambda > 0$). Thus, we can provide good anthropic reasons why we can expect to live in an old universe that is neither too far from flatness nor dominated by a much stronger cosmological constant than observed ($|\Lambda| \leq 10|\Lambda_{\text{obs}}|$). Our results for varying α suggest that there might be significant anthropic constraints if Λ or the spatial curvature is too small to prevent domination before atomic structures become impossible.

3 Observations in Space and in the Lab

Studies of relativistic fine structure in the absorption lines of dust clouds around quasars by Webb et al., [2, 3], have led to widespread theoretical interest in the question of whether the fine structure constant has varied in time. The quasar data analysed in refs. [2, 3] consists of three separate samples of Keck-Hires observations which combine to give a data set of 128 objects at redshifts $0.5 < z < 3$. The many-multiplet technique finds that their absorption spectra are consistent with a shift in the value of the fine structure constant between

these redshifts and the present of $\Delta\alpha/\alpha \equiv [\alpha(z) - \alpha]/\alpha = -0.57 \pm 0.10 \times 10^{-5}$, where $\alpha \equiv \alpha(0)$ is the present value of the fine structure constant [2, 3]. Extensive analysis has yet to find a selection effect that can explain the sense and magnitude of the relativistic line-shifts underpinning these deductions. A smaller study of 23 VLT-UVES absorption systems between $0.4 \leq z \leq 2.3$ by Chand *et al.* [34] initially found $\Delta\alpha/\alpha = -0.6 \pm 0.6 \times 10^{-6}$ by using an approximate version of the full MM technique. However a recent reanalysis of the same data by Murphy *et al.* using the full unbiased MM method increased the uncertainties and suggested the revised figure of $\Delta\alpha/\alpha = -0.64 \pm 0.36 \times 10^{-5}$ for the same data [4].

Any variation of α today can also be constrained by direct laboratory searches. These are performed by comparing clocks based on different atomic frequency standards over a period of months or years. Until very recently, the most stringent constraints on the temporal variation in α arose by combining measurements of the frequencies of Sr [35], Hg+ [36], Yb+ [37], and H [38] relative to Caesium: $\dot{\alpha}/\alpha = (-3.3 \pm 3.0) \times 10^{-16} \text{ yr}^{-1}$. Cingöz *et al.* also recently reported a less stringent limit of $\dot{\alpha}/\alpha = -(2.7 \pm 2.6) \times 10^{-15} \text{ yr}^{-1}$ [39]; however, if the systematics can be fully understood, an ultimate sensitivity of 10^{-18} yr^{-1} is possible with their method [40]. If a linear variation in α is assumed then the Murphy *et al.* quasar measurements equate to $\dot{\alpha}/\alpha = (6.4 \pm 1.4) \times 10^{-16} \text{ yr}^{-1}$. If the variation is due to a light scalar field described by a theory like that of BSBM [15], then the rate of change in the constants is exponentially damped during the recent dark-energy-dominated era of accelerated expansion, and one typically predicts $\dot{\alpha}/\alpha = 1.1 \pm 0.3 \times 10^{-16} \text{ yr}^{-1}$ from the Murphy *et al.* data, which is not ruled out by the atomic-clock constraints mentioned above. For comparison, the Oklo natural reactor constraints, which reflect the need for the $\text{Sm}^{149} + n \rightarrow \text{Sm}^{150} + \gamma$ neutron capture resonance at 97.3 meV to have been present 1.8 – 2 Gyr ($z = 0.15$) ago, as first pointed out by Shlyakhter [24], are currently $\Delta\alpha/\alpha = (-0.8 \pm 1.0) \times 10^{-8}$ or $(8.8 \pm 0.7) \times 10^{-8}$ (because of the double-valued character of the neutron capture cross-section with reactor temperature) and $\Delta\alpha/\alpha > 4.5 \times 10^{-8}$ (6σ) when the non-thermal neutron spectrum is taken into account. However, there remain significant environmental uncertainties regarding the reactor's early history and the deductions of bounds on constants. The quoted Oklo constraints on α apply only when all other constants are held to be fixed. If the quark masses to vary relative to the QCD scale, the ability of Oklo to constrain variations in α is greatly reduced [41].

Recently, Rosenband *et al.* [42] measured the ratio of aluminium and mercury single-ion optical clock frequencies, $f_{\text{Al}^+}/f_{\text{Hg}^+}$, repeated over a period of about a year. From these measurements, the linear rate of change in this ratio was found to be $(-5.3 \pm 7.9) \times 10^{-17} \text{ yr}^{-1}$. These measurements provides the strongest limit yet on any temporal drift in the value of α : $\dot{\alpha}/\alpha = (-1.6 \pm 2.3) \times 10^{-17} \text{ yr}^{-1}$. This limit is strong enough to strongly rule out theoretical explanations of the change in α reported by Webb *et al.* [2, 3] in terms of the slow variation of an effectively massless scalar field, even allowing for the damping by cosmological acceleration, unless there is a significant new physical effect that slows the locally observed effects of changing α on cosmological

scales (for a detailed analysis of global-local coupling of variations in constants see Refs. [43]).

It has been noted that if the ‘constants’ such as α or μ can vary, then in addition to a slow temporal drift one would also expect to see an annual modulation in their values. In many theories, the Sun perturbs the values of the constants by a factor roughly proportional to the Sun’s Newtonian gravitational potential (the contribution from the Earth’s gravitational potential is about 14 times smaller than that of the Sun’s at the Earth’s surface). Hence the ‘constants’ depend on the distance from the Sun. Since the Earth’s orbit around the Sun has a small ellipticity, the distance, r , between the Earth and Sun fluctuates annually, reaching a maximum at aphelion around the beginning of July and a minimum at perihelion in early January. It was shown in Ref. [44] that in many varying constant models, the values of the constants measured here on Earth, would oscillate in a similar seasonal manner. Moreover, in many cases, this seasonal fluctuation is predicted to dominate over any linear temporal drift [44].

Specifically, let us suppose that the Sun creates a distance-dependent perturbation to the measured value of a coupling constant, \mathcal{C} , of amplitude $\delta \ln \mathcal{C} = C(r)$. If this coupling constant is measured on the surface of another body (e.g. the Earth) which orbits the first body along an elliptical path with semi-major axis a , period T_p , and eccentricity $e \ll 1$, then to leading order in e , the annual fluctuation in \mathcal{C} , $\delta \mathcal{C}_{\text{annual}}$ will be given by

$$\frac{\delta \mathcal{C}_{\text{annual}}}{\mathcal{C}} = -c_{\mathcal{C}} \cos\left(\frac{2\pi t}{T_p}\right) + O(e^2), \quad (10)$$

where $c_{\mathcal{C}} \equiv e a C'(a)$, $C'(a) = dC(r)/dr|_{r=a}$ and $t = nT_p$, for any integer n , corresponds to the moment of closest approach (perihelion). In the case of the Earth moving around the Sun, over a period of 6 months from perihelion to aphelion one would therefore measure a change in the constant \mathcal{C} equal to $2c_{\mathcal{C}}$. Using $\delta \ln(f_{\text{Al}+}/f_{\text{Hg}+}) = (3.19 + 0.008)\delta\alpha/\alpha$, [42], a maximum likelihood fit to the data gives $c_{\alpha} = e a \delta\alpha'(a) = (-0.89 \pm 0.84) \times 10^{-17}$, where $a = 149,597,887.5$ km is the semi-major axis of the Earth’s orbit, and $\delta\alpha(r)$ is the perturbation in α due to the Sun’s gravitational field. Assume that over solar system scales, the values of the scalar fields on which values of the ‘constants’ depend, vary with the local gravitational potential. Hence, we have $\delta\alpha(r)/\alpha = k_{\alpha} \Delta U_{\odot}(r)$, where k_{α} is a theory-dependent multiplier, ΔU_{\odot} is the change in the gravitational potential of the Sun: $U_{\odot}(r) = -GM_{\odot}/r$, and so $e a \Delta U'_{\odot}(a) = eGM_{\odot}/a = 1.65 \times 10^{-10}$. Hence, we find:

$$k_{\alpha} = (-5.4 \pm 5.1) \times 10^{-8}. \quad (11)$$

The frequency shifts measured by Rosenband *et al.* [42] were not sensitive to changes in the electron-proton mass ratio: $\mu = m_e/m_p$. Measurements of optical transition frequencies relative to Cs, Refs. [35, 36, 37, 38], are sensitive to both μ and α . H-maser atomic clocks [45] are also sensitive to variations in the light quark to proton mass ratio: $q = m_q/m_p$. We can use all these observations if we define two more gravitational coupling multipliers, k_{μ} and k_q ,

by $\delta\mu/\mu = k_\mu\Delta U_\odot$, and $\delta q/q = k_q\Delta U_\odot$. Refs. [35, 36, 45] give $k_\alpha + 0.36k_\mu = (-2.1 \pm 3.2) \times 10^{-6}$, $k_\alpha + 0.17k_\mu = (3.5 \pm 6.0) \times 10^{-7}$, and $k_\alpha + 0.13k_q = (1 \pm 17) \times 10^{-7}$ respectively. We also performed a bootstrap seasonal fluctuation fit (with 10^5 resamplings) to the Yb^+ frequency measurements of Peik *et al.* [37] giving $k_\alpha + 0.51k_\mu = (7.1 \pm 3.4) \times 10^{-6}$ [?]. Combining these bounds with Eq. (11) gives $k_\mu = (3.9 \pm 3.1) \times 10^{-6}$, $k_q = (0.1 \pm 1.4) \times 10^{-5}$. Recently, Blatt *et al.* [35] combined data from measurements of H-maser [45] and optical atomic clocks [35, 36], to bound the multipliers, k_α , k_μ and k_q , finding $k_\alpha = (2.5 \pm 3.1) \times 10^{-6}$. The constraint on k_α derived in this paper from the data of Rosenband *et al.* [42] therefore represents an improvement by about two orders of magnitude over the previous best bound. This improved bound on k_α combined with data found by Peik *et al.* [37] has also produced an order of magnitude improvement in the determination of k_μ and a slight improvement in the constraint on k_q .

Seasonal fluctuations are predicted by a varying constant theory because the scalar field which drives the variation in the constant couples to normal matter. The presence of the Sun therefore induces gradients in scalar fields and associated varying 'constants', and it is essentially these gradients that are detectable as seasonal variables. As mentioned earlier, gradients in a scalar field which couples to normal matter result in new or 'fifth' forces with pseudo-gravitational effects. In the case of varying α and μ theories, these forces are almost always composition dependent, which would violate the universality of free-fall and hence the weak equivalence principle (WEP). The magnitude of any composition-dependent fifth force toward the Sun is currently constrained to be no stronger than $10^{-12} - 10^{-13}$ times than the gravitational force [46]. In the context of a given theory the constraints from WEP tests indirectly bound k_α . Indeed, they often provide the tightest constraints on k_α [?, 47, 48].

A recent thorough analysis of the WEP violation constraints on k_α [47, 49] found $k_\alpha = (0.3 \pm 1.7) \times 10^{-9}$, with a similar constraint on k_q . It must be noted, however, that this result is still subject to theoretical uncertainty, especially regarding the dependence of nuclear properties on quark masses. For instance, it was also noted in Ref. [47] that if certain (fairly reasonable) assumptions about nuclear structure are dropped, the 1σ error bars on k_α increase by about an order of magnitude to: $\pm 1.4 \times 10^{-8}$. Despite these uncertainties, for many theories of varying α , WEP violation constraints from laboratory experiments or lunar laser ranging [50] still provide the strongest, albeit indirect, bound on k_α .

4 Conclusions

We have described some of the history of the study of varying constants in physics. This area of research has been reinvigorated by a significant body of observational data, drawn from quasar absorption spectra, which is consistent with a change in the value of the fine structure constant, α , over of a few parts in a million over 10 billion years. So far, these data have neither been reliably

confirmed nor contradicted by other observational studies and this confrontation is keenly awaited. We described how a simple self-consistent theory of varying α developed by Sandvik, Barrow and Magueijo can be constructed and the clear pattern of variation that it predicts in the universe: no variation of α during the radiation era and a logarithmic time increase during the cold-dark-matter era, followed by a resumption of no time variation in α after the universe begins to accelerate during the dark-energy era. A fit of these simple models to the observational data fixes the one free parameter defining the theory and predicts a violation of the weak equivalence principle at a 10^{-13} level that is easily detectable from space. We also described exciting new developments in the laboratory search for varying α . These experiments are for the first time achieving the sensitivities of the indirect astronomical bounds and in the next few years we may be able to draw some strong conclusions about varying constants from the confluence of laboratory experiments and large new astronomical data sets.

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References

- [1] J.D. Barrow, *The Constants of Nature*, Cape, London (2002).
- [2] M. Murphy, J. Webb, V. Flambaum, V. Dzuba, C. Churchill, J. Prochaska, A. Wolfe, Mon. Not. R. astr. Soc., 327, 1208 (2001)
- [3] J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater J.D. Barrow, Phys. Rev. Lett. **82**, 884 (1999); J.K. Webb, M.T. Murphy, V.V. Flambaum, V.A. Dzuba, J.D. Barrow, C.W. Churchill, J.X. Prochaska, A.M. Wolfe, Phys. Rev. Lett. **87**, 091301 (2001).
- [4] M. Murphy, J.K. Webb & V. Flambaum, Phys. Rev. Lett. 99, 239001,(2007) and astro-ph/0911.4512
- [5] E. A. Milne, *Relativity, Gravitation and World Structure*, Clarendon, Oxford, (1935).
- [6] J.B.S. Haldane, Nature 139, 1002 (1937) and 158, 555 (1944).
- [7] H. Weyl, Ann. Physik 59, 129 (1919).
- [8] J.D. Barrow & F.J. Tipler, *The Anthropic Cosmological Principle*, OUP, Oxford (1986).
- [9] P.A.M. Dirac, Nature 139, 323 (1937).
- [10] E. Teller, Phys. Rev. 73, 801 (1948).
- [11] R.H. Dicke, Rev. Mod. Phys. 29, 355 (1957) and Nature 192, 440 (1961).

- [12] G. Farmelo, *The Strangest Man*, Faber & Faber, London (2009), p221.
- [13] C. Brans & R.H. Dicke, Phys. Rev. 124, 924 (1961).
- [14] J.D. Bekenstein, Phys. Rev. D 25, 1527 (1982).
- [15] H. B. Sandvik, J. D. Barrow & J. Magueijo, Phys. Rev. Lett. 88, 031302 (2002), Phys. Rev. D 65, 063504 (2002), Phys. Rev. D 65, 123501 (2002), Phys. Rev. D 66, 043515 (2002), Phys. Lett. B 541, 201 (2002), J.D. Barrow & D. Mota, Class.Quant.Grav. 19, 6197 (2002).
- [16] D. Kimberly & J. Magueijo, Phys. Lett. B 584, 8 (2004)
- [17] D. Shaw & J.D. Barrow, gr-qc/0412135.X. Calmet & H. Fritzsch, hep-ph/0112110.
- [18] G. Gamow, Phys. Rev. Lett. 19, 759 (1967).
- [19] K.P. Stanyukovich, Sov. Phys. Dokl. 7, 1150 (1963).
- [20] F. Dyson, Phys. Rev. Lett. 19, 1291 (1967).
- [21] A. Albrecht & J. Magueijo, Phys. Rev. D 59, 043516 (1999).
- [22] S. Landau, P. Sisterna & H. Vucetich, Phys. Rev. D 63, 081303(R) (2001).
- [23] G. Dvali & M. Zaldarriaga, hep-ph/0108217.
- [24] A. Shylakhter, Nature 264, 340 (1976). Y. Fujii et al, Nucl. Phys. B, 573, 377 (2000); T. Damour & F.J. Dyson, Nucl. Phys. B, 480, 37 (1996).
- [25] R. Battye, R. Crittenden, J. Weller, Phys. Rev. D 63, 043505.
- [26] P.P. Avelino et. al., Phys. Rev. D 62, 123508, (2000) and astro-ph/0102144 (2001), J.D. Barrow & D. Mota, Class.Quant.Grav. 19, 6197 (2002).
- [27] J. Magueijo, J.D. Barrow & H. Sandvik, astro-ph/0202374.
- [28] T. Damour & A. Polyakov, Nucl. Phys. B 423, 532 (1994).
- [29] G.R. Dvali & M. Zaldarriaga, Phys.Rev.Lett. 88 (2002) 091303.
- [30] T. Damour, F. Piazza, & G. Veneziano, Phys. Rev. D 66, 046007 (2002) and Phys. Rev. Lett. 89, 081601 (2002).
- [31] T.J. Sumner et al, Adv. in Space Research, 39, 254 (2007).
- [32] B. Carter, in *Confrontation of Cosmological Theories with Observation*, ed., M.S. Longair, Reidel, Dordrecht, (1974), p291 and *Large Numbers in Astrophysics and Cosmology*, unpublished preprint, Inst. Theoretical Astronomy, Cambridge Univ., (1968).
- [33] C.B. Collins & S.W. Hawking, Astrophys. J. 180, 317 (1973).

- [34] H. Chand *et al.*, *Astron. Astrophys.* 417, 853 (2004); R. Srianand *et al.*, *Phys. Rev. Lett.* **92**, 121302 (2004).
- [35] S. Blatt *et al.*, *Phys. Rev. Lett.* 100, 140801 (2008).
- [36] T. M. Fortier *et al.*, *Phys. Rev. Lett.* 98, 070801 (2007).
- [37] E. Peik *et al.*, *Phys. Rev. Lett.* 93, 170801 (2004); E. Peik *et al.*, to appear in *Proc. 11th Marcel Grossmann Meeting, Berlin, 2006*, ed. R. Jantzen, World Scientific: Singapore, (2008), arXiv:physics/0611088.
- [38] M. Fischer *et al.*, *Phys. Rev. Lett.* 92, 230802 (2004).
- [39] A. Cingöz *et al.*, *Phys. Rev. Lett.* 98, 040801 (2007).
- [40] A. T. Nguyen *et al.*, *Phys. Rev. A* 69, 022105 (2004).
- [41] V. V. Flambaum, in *Proc. Festschrift in Honor of Bruce H J McKellar and Girish C Joshi, Melbourne, 2006*, ed. R. R. Volkas, B. H. J. McKellar, G. C. Joshi, World Scientific: Singapore, (2007).
- [42] T. Rosenband *et al.*, *Science* 319, 1808 (2008).
- [43] D.J. Shaw & J.D. Barrow, *Phys. Rev. D* 71, 063525 (2005); *Phys. Rev. D* 73, 123505 (2006); *Phys. Rev. D* 73, 123506 (2006).
- [44] D.J. Shaw & J.D. Barrow, *Phys. Rev. D* 78, 067304, (2008).
- [45] N. Ashby *et al.*, *Phys. Rev. Lett.* 98, 070802 (2007).
- [46] S. Schlamminger *et al.*, *Phys. Rev. Lett.* 100, 041101 (2008); S. Baessler *et al.*, *Phys. Rev. Lett.* 83 3585 (1999); Y. Su *et al.*, *Phys. Rev. D* 50, 3614 (1994); V. G. Braginsky & V. I. Panov, *Sov. Phys. JETP* 34, 463 (1972).
- [47] T. Dent (2008), arXiv:0805.0318
- [48] J.D. Barrow & J. Magueijo, *Phys. Rev. D* 72, 043521 (2005); J.D. Barrow, *Phys. Rev. D* 71, 083520 (2005).
- [49] T. Dent (2008), Private communication.
- [50] K. Nordtvedt, *Phys. Rev. D* 169, 1014 & 1017 (1968); D.F. Bartlett & D. van Buren, *Phys. Rev. Lett.* 57, 21 (1986).